

## Modulational Instability of Ion-Acoustic Waves in a Plasma with Warm Ions, Nonthermal Electrons and Nonthermal Positrons

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### ABSTRACT

The modulational instability and envelope solitons of ion-acoustic waves have been investigated in an unmagnetized collisionless warm electron–positron–ion (e-p-i) plasma with both the electrons and positrons having nonthermal distribution. Using standard multiple scale perturbation technique a nonlinear Schrödinger equation (NLSE) governing nonlinear evolution of the wave envelope has been derived. Using this equation we have shown through numerical analysis that the presence of nonthermal electrons and positrons plays significant role on the formation and the structure of ion-acoustic envelope solitons.

**Keywords:** Nonthermal electrons, Nonthermal positrons, Nonlinear Schrödinger equation,

### INTRODUCTION

The study of nonlinear wave propagation in electron-positron-ion (e-p-i) plasma has attracted the attention of many researchers because of its relevance to many astrophysical and laboratory environments [1-9]. The nonlinear propagation of different wave modes in e-p-i plasmas has been investigated by a number of authors [10-14]. In most of these studies electrons and positrons are assumed to obey Maxwellian distribution. In plasma where nonequilibrium stationary states exist, Maxwellian distribution might be inadequate for the description of such plasmas. Numerous studies and in-situ measurements on space environment clearly indicate the wide-spread presence of non-Maxwellian populations [15, 16]. In the radiation belts of earth and Jupiter, significant fluxes of energetic particles are found to coexist with thermalized plasma populations. Earlier Vela satellite [17] detected nonthermal population in the Earth's bow-shock region and Scarf et al [18] observed nonthermal electrons and high frequency waves in the upstream solar wind. Existence of non-Maxwellian particle distributions in the astrophysical context has been confirmed by many scientists [19-27]. Moreover other measurements of plasma sheet electron and ion distributions [28, 27], observations in the earth's foreshock [30], observations of electron distribution in Saturn's magnetosphere [31], all come into sight to validate the occurrence of non-Maxwellian distribution in astrophysical and space plasmas. Since it has been confirmed that in space environment the existence of nonequilibrium plasma is more realistic than the equilibrium one, so in recent years a great deal of attention has been given to the study of nonlinear wave propagation in plasmas with different non-Maxwellian particle distributions. Similar to Vella satellite [17], ASPERA on the Phobos 2 satellite [32] also observed that density of highly energetic particles are dropping significantly in the upper Martian ionosphere, which points towards the existence of nonthermal electron fluxes. Density depression in the earth magnetosphere has been observed by Freja

[33] and Viking [34] satellites. Electron density depletion in night side auroral zone was confirmed by Persoon et al [35] too. Simple electron-ion thermal plasma supports only compressive solitary waves but no rarefactive ones. Cairns et al [36] considered a typical plasma model having nonthermal electrons and cold ions and shown that it is possible to obtain both the compressive and rarefactive solitary waves which supports the observations of Freja [33] and Viking [34] satellites. Inspired by the success of Cairns [36] nonthermal model many researchers have studied nonlinear propagation of ion-acoustic solitary waves in e-p-i plasma with nonthermal electrons [11, 14, 37-45]. In most of these studies electrons are assumed to be nonthermal while positrons are assumed to be Maxwellian. Electrons and positrons have the same mass and charge of equal magnitude with opposite sign. In astrophysical and space plasmas electrons and positrons are produced under identical environments and it is expected that similar to electrons, positrons can also have non-Maxwellian distribution. Positrons are produced by pair production in high energy processes occurring in many astrophysical environments. Also, positrons produced in laser plasma experiments may get trapped before they finally thermalize in plasma. There are many such sources that point towards the simultaneous existence of both nonthermal electrons and nonthermal positrons. So it would be of practical interest to study nonlinear wave propagation in e-p-i plasma considering both the electrons and positrons to be non-Maxwellian.

Recently a few authors have considered non-Maxwellian distribution for both the electrons and positrons in e-p-i plasma and studied KdV like solitary structures [46 - 48]. Williams and Kourakis [46] have investigated ion-acoustic-waves in a magnetized e-p-i plasma assuming kappa distribution for both the electrons and positrons. They have shown that as the value of spectral index becomes smaller which indicates increase in superthermality leads to lower amplitudes and narrower solitons.

Arshad et al [47] have studied kinetic instability of IAWs in magnetospheric e-p-i plasma with kappa distributed electrons and kappa distributed positrons. They have shown that spectral index plays an important role on the stability of the wave. Very recently Ghosh and Bannerjee [48] have investigated ion-acoustic solitary waves in e-p-i plasma with both the electrons and positrons having nonthermal Cairn's type distribution. They have shown that the nonthermality of electrons and positrons plays significant role on the formation and nature of solitary waves. Actually these high energetic non-Maxwellian distributions appear to be more appropriate than a Maxwellian distribution in a wide range of space, astrophysical and laboratory plasma environments. Most of the works cited above are confined to the study of KdV type solitary wave structures. However, the nonlinear propagation of waves in a dispersive medium is generically subject to amplitude modulation due to carrier wave self-interaction caused by intrinsic nonlinearity in the medium. A balance between nonlinearity and group velocity dispersion leads to the formation of envelope solitary structure. Very recently the modulational instability of ion-acoustic waves in e-p-i plasma has been studied by a few authors [42, 47]. However these authors have considered nonthermal distribution for electrons but Maxwellian distribution for positrons. To the best of our knowledge modulational instability of ion-acoustic waves in e-p-i plasma has not so far been reported by anyone considering nonthermal Cairns type distribution for both the electrons and positrons. The purpose of the paper is to address this problem. Here we have investigated the modulational instability of ion-acoustic waves in warm e-p-i plasma considering Cairns type nonthermal distribution for both the electrons and positrons. Using the standard multiple scale perturbation technique a nonlinear Schrödinger (NLS) equation has been derived. Using this equation we have studied the effect of nonthermality of electrons and positrons on the amplitude modulation of ion-acoustic waves in e-p-i plasma. It is shown that the electron- positron nonthermality has significant effect on modulational instability and structure of envelope solitons.

## GOVERNING EQUATIONS AND DERIVATION OF NLSE

We consider an unmagnetized collisionless plasma consisting of nonthermal electrons, nonthermal positrons and warm positive ions. The normalized basic equations governing ion dynamics in one dimension are the following [14, 48]:

$$\begin{aligned}\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) &= 0 \\ \frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{3\sigma_i}{(1-\chi)^2} n_i \frac{\partial n_i}{\partial x} &= -\frac{\partial \phi}{\partial x} \\ \frac{\partial^2 \phi}{\partial x^2} &= n_e - n_p - n_i\end{aligned}$$

Here  $n_i$  and  $v_i$  are respectively the concentration and velocity of the positive ions;  $n_e$  and  $n_p$  are respectively the concentration of electrons and positrons;  $\phi$  is the electrostatic potential, other parameters have their usual significance; the velocities are

normalized by ion-acoustic speed  $C_s = \sqrt{\frac{k_B T_e}{m_i}}$ ; all densities are normalized by equilibrium electron density  $n_{e0}$ ; all length  $x$  by the

electron Debye length  $\lambda_{De} = \sqrt{\frac{k_B T_e}{4e^2 n_{e0}}}$ ; time by  $\frac{\lambda_{De}}{C_s}$ ; ion temperature

by  $T_i$  by  $T_e$  and the electrostatic potential  $\phi$  by  $\frac{k_B T_e}{e}$ ; where  $k_B$  is the Boltzmann's constant and  $\sigma_i = T_i / T_e$ . The nonthermal electron density is given by [36]:

$$n_e = (1 - \beta_e \phi + \beta_e \phi^2) \exp(\phi) \quad (5)$$

where  $\beta_e = \frac{4\delta}{1+3\delta}$  represents the nonthermality of electron distribution and  $\delta$  determines the presence of nonthermal electrons in the plasma. Note that Eq. (5) expresses the isothermal electron

distribution when  $\beta_e = 0$ . The density of nonthermal positrons is given by

$$n_p = \chi (1 + \beta_p \sigma_p \phi + \beta_p \sigma_p^2 \phi^2) \exp(-\sigma_p \phi) \quad (6)$$

where  $\chi = n_{p0} / n_{e0}$  is the ratio between the unperturbed positron and electron number densities and  $\sigma_p = T_e / T_p$  is the ratio between electron and positron temperatures. The equilibrium charge neutrality condition is given by

$$\chi + n_{i0} = 1 \quad (7)$$

Using Eqs. (5) and (6), Eq. (4) can be re-written as

$$\begin{aligned}\frac{\partial^2 \phi}{\partial x^2} &= (1 - \beta_e + \beta_e \phi^2) \exp(\phi) - \\ &\chi (1 + \beta_p \sigma_p \phi + \beta_p \sigma_p^2 \phi^2) \exp(-\sigma_p \phi) - n_i\end{aligned} \quad (8)$$

In order to study the nonlinear evolution of the wave we make the following Fourier expansions for the field quantities [49]

$$F = \varepsilon^2 F_0' + \sum_{s=1}^{\infty} \varepsilon_s \{F_s \exp(is\psi) + F_s^* \exp(-is\psi)\} \quad (9)$$

where  $F$  stands for the field quantities  $n_i$ ,  $v_i$  and  $\phi$ ;  $F_0'$  and  $F_s$  are assumed be function  $\xi$  and  $\tau$  where,  $\xi = \varepsilon(x - C_g t)$  and  $\tau = \varepsilon^2 \tau$ ; here  $\varepsilon$  is a small parameter measuring weakness of

dispersion and nonlinearity.  $C_g = \frac{d\omega}{dk}$  is the group velocity and  $\psi = kx - \omega t$  ( $\omega$  and  $k$  satisfy the linear dispersion relation). Substituting the expansion (9) in Eqs. (1) - (3) and (8) and then equating from both sides the coefficients of  $\exp(i\psi)$ ,  $\exp(2i\psi)$  and terms independent of  $\psi$  we obtain three sets of equations which we call respectively I, II and III. To solve these equations we make

the following perturbation expansion for the field quantities  $F_0'$  and  $F_s$ :

$$X = X^{(1)} + \varepsilon X^{(2)} + \varepsilon X^{(3)} + \dots \quad (10)$$

Solving the lowest order equations obtained from the set of equations I after substituting the expansion (10) we get the following solutions for the first harmonic quantities in the lowest order:

$$n_{i1}^{(1)} = (X_1 + k^2)\alpha$$

$$v_{i1}^{(1)} = \frac{\omega}{k(1-\chi)}(X_1 + k^2)\alpha$$

where  $\alpha = \phi_i^{(1)}$   
and

$$X_1 = (1 - \beta_e) + \chi\sigma_p(1 - \beta_p)$$

The linear dispersion relation is

$$\omega^2 = k^2 \left[ \frac{(1-\chi)}{(X_1 + k^2)} + \frac{3\sigma_i}{(1-\chi)} \right] \quad (14)$$

Fig. 1 shows dispersion relation for different values of the nonthermal positron parameter ( $\beta_p$ ). It shows that the wave frequency increases with increase in the value of  $\beta_p$ .

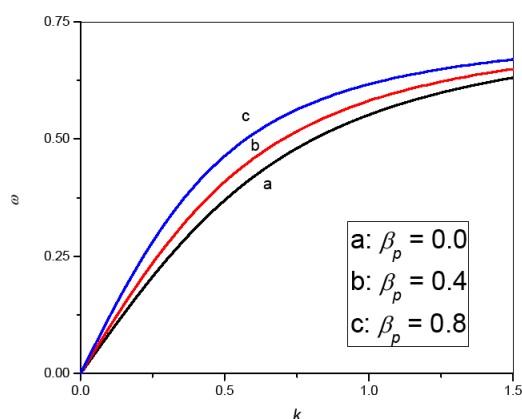


Fig.1.  $\omega$  versus wavenumber  $k$  for different values of nonthermal parameter ( $\beta_p$ ). Curves labelled a, b and c correspond to  $\beta_p = 0, 0.4$  and  $0.8$  respectively.  $\chi = 0.5$ ,  $\sigma_p = 0.6$ ,  $\sigma_i = 0.001$  and  $\beta_e = 0.5$ .

Note that with  $\beta_p = \beta_e = 0, \chi = 0, \sigma_i = 0$  the linear dispersion relation (14) reduces to that obtained by Kakutani and Sugimoto [50] for ion-acoustic waves in e-i plasma.

The group velocity ( $C_g$ ), of the wave is given by:

$$C_g = \frac{d\omega}{dk} = \frac{k}{\omega} \left[ \frac{(1-\chi)}{(X_1 + k^2)} - \frac{k^2(1-\chi)}{(X_1 + k^2)^2} + \frac{3\sigma_i}{(1-\chi)} \right] \quad (15)$$

First harmonic quantities in the second order can be obtained by substituting the perturbation expansion (10) in the set of equations

I and solving order  $\epsilon^2$  equations. Thus we obtain,  $\phi_1^{(2)} = 0$

$$n_{i1}^{(2)} = -i2k \frac{\partial \alpha}{\partial \xi}$$

$$v_{i1}^{(2)} = -i \left[ \frac{2\omega}{(1-\chi)} + \frac{(X_1 + k^2)}{k(1-\chi)} \left\{ C_g - \frac{\omega}{k} \right\} \right] \frac{\partial \alpha}{\partial \xi} \quad (16)$$

The second harmonic quantities in the lowest order obtained from the set of equations II after substituting the expansion (10) are the following:

$$\phi_2^{(1)} = \frac{X_3}{X_2} \alpha^2 \quad (11)$$

$$n_{i2}^{(1)} = (X_1 + 4k^2) \phi_2^{(1)} + (1 - \chi\sigma_p^2) \alpha^2 \quad (12)$$

$$v_{i2}^{(1)} = \frac{\omega(X_1 + 4k^2)}{k(1-\chi)} \phi_2^{(1)} - \frac{\omega}{k(1-\chi)} \left[ \frac{(X_1 + k^2)^2}{(1-\chi)} - (1 - \chi\sigma_p^2) \right] \alpha^2 \quad (13)$$

where,

$$X_2 = \frac{2\omega^2(X_1 + 4k^2)}{k(1-\chi)} - 2k - \frac{6\sigma_i k(X_1 + 4k^2)}{(1-\chi)^2} \quad (14)$$

and

$$X_3 = \frac{3\omega^2(X_1 + k^2)^2}{k(1-\chi)^2} + \frac{2(1-\chi\sigma_p^2)}{(1-\chi)} \left[ \frac{3\sigma_i k}{(1-\chi)} \right] + \frac{3\sigma_i k(X_1 + k^2)^2}{(1-\chi)^2} - \frac{2\omega^2(1-\chi\sigma_p^2)}{k(1-\chi)} \quad (15)$$

The zeroth harmonic components generated through nonlinear self-interaction of the finite amplitude wave are obtained from the set of equations III after substituting the expansion (10):

$$\phi_0^{(1)} = \frac{X_5}{X_4} \alpha \alpha^*$$

$$n_0^{(1)} = \left[ \frac{X_1 X_5}{X_4} + (1 - \chi\sigma_p^2) \right] \alpha \alpha^*$$

$$v_0^{(1)} = \left[ \frac{C_g}{(1-\chi)} \left\{ (1 - \chi\sigma_p^2) + \frac{X_1 X_5}{X_4} \right\} - \frac{2\omega(X_1 + k^2)^2}{k(1-\chi)^2} \right] \alpha \alpha^* \quad (16)$$

where,

$$X_4 = \frac{C_g^2 X_1}{(1-\chi)} - \frac{3\sigma_i X_1}{(1-\chi)^2} - 1 \quad (17)$$

and

$$X_5 = \frac{(X_1 + k^2)^2}{(1-\chi)^2} \left\{ \frac{2C_g \omega}{k} + \frac{\omega^2}{k^2} + 3\sigma_i \right\} + \frac{(1 - \chi\sigma_p^2)}{(1-\chi)} \left[ \frac{3\sigma_i}{(1-\chi)} - C_g^2 \right] \quad (18)$$

Now to derive the desired NLS equation we require first harmonic quantities in the third order. Collecting coefficients of  $\epsilon^3$  from both sides of the set of equations I after substituting the perturbation expansion (10) we get a set of equations for first harmonic quantities in the third order from which after proper elimination we obtain the following NLS equation:

$$i \frac{\partial \alpha}{\partial \tau} + P \frac{\partial^2 \alpha}{\partial \xi^2} = Q \alpha^2 \alpha^* \quad (19)$$

where,

$$P = \frac{k(1-\chi)}{2\omega(X_1 + k^2)} X_6 \quad (20)$$

and

$$Q = \frac{k(1-\chi)}{2\omega(X_1+k^2)} [X_7] \quad (25)$$

in which,

$$X_6 = \left\{ \frac{2\omega}{(1-\chi)} + \frac{(X_1+k^2)}{k(1-\chi)} \left( C_g - \frac{\omega}{k} \right) \right\} \left( \frac{\omega}{k} - C_g \right) - \frac{\omega}{(1-\chi)} \left( \frac{\omega}{k} + 2C_g \right) + \frac{9\sigma_i k}{(1-\chi)^2} \quad (26)$$

$$X_7 = \omega f_1 + k f_2 - \frac{(1-\chi\sigma_p^2)}{(1-\chi)} f_3 \left( \frac{\omega^2}{k} - \frac{3\sigma_i k}{(1-\chi)} \right) \quad (27)$$

and

$$f_1 = \left\{ \frac{X_1 X_5}{X_4} + (1-\chi\sigma_p^2) \right\} \left( \frac{\omega}{k} + C_g \right) \cdot \frac{(X_1+k^2)}{(1-\chi)} - \frac{3\omega(X_1+k^2)^3}{k(1-\chi)^2} + \frac{2\omega(X_1+k^2)(X_1+4k^2)}{k(1-\chi)} \cdot \frac{X_3}{X_2} \quad (28)$$

$$f_2 = \left( \frac{\omega C_g}{k} + 3\sigma_i \right) \frac{(X_1+k^2)}{(1-\chi)^2} \cdot \left\{ \frac{X_1 X_5}{X_4} + (1-\chi\sigma_p^2) \right\} - \frac{3\omega^2(X_1+k^2)^3}{k^2(1-\chi)^3} + \left( \frac{\omega^2}{k^2} + 3\sigma_i \right) \frac{(X_1+k^2)}{(1-\chi)^2} \cdot \left\{ (X_1+4k^2) \cdot \frac{X_3}{X_2} + (1-\chi\sigma_p^2) \right\} \quad (29)$$

and

$$f_3 = \left\{ \frac{X_3}{X_2} + \frac{X_5}{X_4} \right\} \cdot (1-\chi\sigma_p^2) \quad (30)$$

## RESULTS AND DISCUSSION

The NLS equation (23) describes the nonlinear evolution of the amplitude of IAWs in e-p-i plasma with nonthermal electrons, nonthermal positrons and warm positive ions. The NLS equation (23) has been studied in details by many authors in connection with the nonlinear propagation of different wave modes in plasma. By considering a small perturbation to the stationary plane wave solution of the NLS equation, it can be shown that the wave is modulationally stable for  $PQ > 0$  and unstable for  $PQ < 0$ . Eqs. (24) and (25) show that the coefficients of dispersion  $P$  and nonlinearity  $Q$  are all dependent on various plasma parameters such as nonthermal parameters of electrons and positrons, positron concentration, ion temperature, etc. So the product  $PQ$  can have both positive and negative values in different parametric regions. In the unstable region the growth rate of instability is shown to attain a maximum value for certain wavenumber of perturbation and the corresponding maximum growth rate is given

by  $g_m = |Q|\alpha_0^2$  where  $\alpha_0$  is the normalized carrier wave amplitude. In order to investigate the stability / instability profile of the wave we have numerically calculated the ratio  $P/Q$  (which is related to the width of the possible envelope soliton structures) over a wide range of plasma parameters. For a given set of plasma parameters the wave is shown to be modulationally stable ( $PQ > 0$ ) for low values of wavenumber and modulationally unstable ( $PQ < 0$ ) for wavenumber exceeding certain critical value  $k_c$ .

Also we have numerically examined different parametric regions of stability and instability. As the coefficients  $P$  and  $Q$  depend on nonthermal parameters  $\beta_e$  and  $\beta_p$ , these parameters would certainly play crucial role and affect modulational instability and the

excitation of envelope solitons. Numerical plots in Figs. 2 and 3 show  $P/Q$  as a function of  $k$  for different values of  $\beta_e$  and  $\beta_p$  respectively. It shows that the IAWs remain modulationally stable for  $k$  less than certain critical value  $k_c$  and for  $k > k_c$  the wave is modulationally unstable. Numerically we find that for  $\beta_e = \beta_p = 0, \chi = 0, \sigma_i = 0$  the critical value of  $k$  is 1.47 which is in good agreement

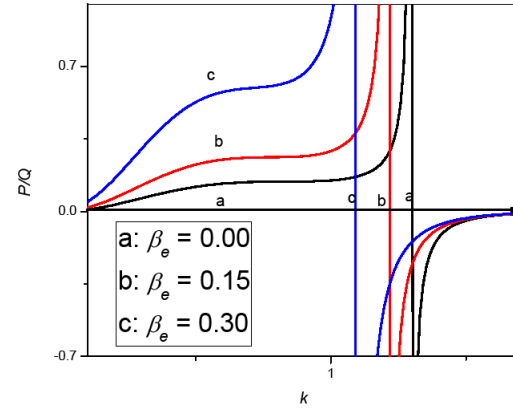


Fig. 2 . Plot of  $P/Q$  versus wave number  $k$  for different values of nonthermal parameter ( $\beta_e$ ). Curves labelled a, b and c correspond to  $\beta_e = 0, 0.15$  and  $0.30$  respectively.  $\chi = 0.32, \sigma_p = 0.4, \sigma_i = 0.005$  and  $\beta_p = 0.2$ .

with results of Kakutani and Sugimoto [50]. For the unstable wave packet ( $PQ < 0$ ), it can be shown that the ion-acoustic wave propagate as an envelope soliton called bright soliton. On the other hand, for stable wave packet ( $PQ > 0$ ), the wave can propagate in the form of an envelope hole called dark soliton. The soliton width is determined by the ratio  $|P/Q|$ .

In Fig. 2 we show the variation of  $P/Q$  with wavenumber for different values of the electron nonthermal parameter ( $\beta_e$ ), keeping positron concentration ( $\chi$ ), ion-temperature ( $\sigma_i$ ) and  $\beta_p$  fixed. It shows that as  $\beta_e$  increases, the value of critical wavenumber separating stable and unstable regions decreases. Qualitatively this result agrees with those obtained by Ghosh et al [42] and Gill et al [45]. Fig. 3 shows the variation of  $P/Q$  with wavenumber for different values of positron nonthermal parameter ( $\beta_p$ ), keeping positron concentration ( $\chi$ ), ion-temperature ( $\sigma_i$ ) and  $\beta_e$  fixed. It shows that as  $\beta_p$  increases, the value of critical wavenumber separating stable and unstable regions decreases.

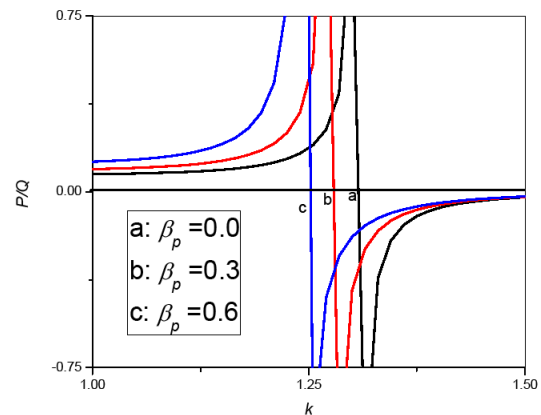




Fig. 3. Plot of  $P/Q$  versus wave number  $k$  for different values of nonthermal parameter ( $\beta_p$ ). Curves labelled a, b and c correspond to  $\beta_p = 0, 0.3$  and  $0.60$  respectively.  $\chi = 0.45$ ,  $\sigma_p = 0.55$ ,  $\sigma_i = 0.005$  and  $\beta_e = 0.1$ .

From Figs. 2 and 3 we can conclude that the width of dark solitons increases and that of the bright solitons decreases with increase in the nonthermal parameters  $\beta_e$  and  $\beta_p$ . In Fig. 4 we show the dependence of instability growth rate on the nonthermal parameter taking  $\sigma_p$  as a parameter. We find that with increase in  $\sigma_p$  (i.e., with increase in electron temperature or decrease in positron temperature), the growth rate decreases. The growth rate of instability decreases with decrease in the nonthermal parameter. This means that the growth rate of modulational instability decreases as the electrons and positrons evolve towards their thermodynamic equilibrium.

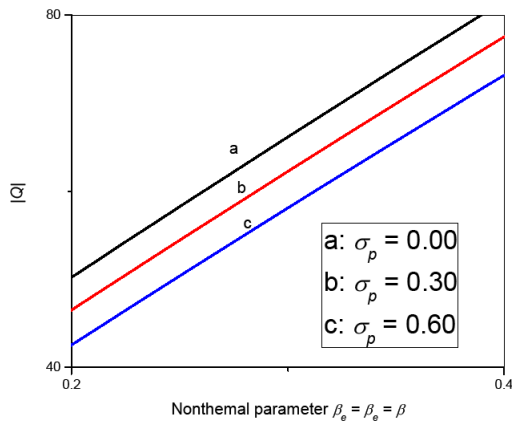


Fig. 4. Dependence of instability growth rate on nonthermal parameter ( $\beta_p = \beta_e = \beta$ ) for different values of  $\sigma_p$ . Curves labelled a, b and c correspond to  $\sigma_p = 0, 0.3$  and  $0.60$  respectively.  $\chi = 0.38$ ,  $k = 1.3$  and  $\sigma_i = 0.017$ .

## SUMMARY AND CONCLUDING REMARKS

In the present work we have investigated the problem of modulational instability of IAWs in warm e-p-i plasma considering simultaneous presence of nonthermal electrons and nonthermal positrons. Our main findings are summarized below:

- (i) The wave frequency increases with increase in nonthermality of both the electrons and positrons.
- (ii) There exists a critical wave number  $k_c$  below which the wave is modulationally stable and above which the wave is modulationally unstable.
- (iii) The critical wavenumber decreases with increase in nonthermality of both the electrons and positrons.
- iv) Dark and bright envelope solitons can be excited in all cases. The amplitude and width of these solitons depend significantly on the nonthermal parameters.
- (v) The growth rate of modulational instability decreases as the electrons and positrons evolve towards their thermodynamic equilibrium (i. e with decrease in nonthermal parameter).

Finally we would like to mention that the results obtained in this paper may be useful to explain modulational instability and envelope soliton excitations of IAWs in some astrophysical and space environments where e-p-i plasmas can be found with nonthermal electrons and nonthermal positrons.

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